Compositional Systems in Finite-Dimensional Classical Mechanics: A Typed Reconstruction and Its Failure Modes

PRINCIPIA PHYSICA Pilot — Topic 1

16 August 2026

Abstract

We give a typed reconstruction of *system*, *state*, *observable*, *process* and *composition* for finite-dimensional classical mechanics, testing — not assuming — the thesis that *physics is the study of empirically realized compositional mathematical structures*. The symplectic product is monoidal but not cartesian: no projection, no diagonal. Centrally, "these two subsystems interact" is *not* an invariant of (M, ω, H), so compositional structure is additional data fixed empirically by what an apparatus reads. And the symplectic Hamiltonian systems are not closed under the operations physicists call composition, the closed class being the Dirac / port-Hamiltonian one. Every claim carrying epistemic status is typed by one of twelve labels, which share a counter with the Counterexample and Remark environments.

1 Scope and epistemic conventions

This paper does not presuppose the pilot thesis, "*physics is the study of empirically realized compositional mathematical structures*"; it builds the smallest compositional apparatus adequate to finite-dimensional classical mechanics, then attacks it. Field theory, general relativity and quantum theory appear only as *boundary tests*. It is Paper I of three, with Paper II, *Hamiltonian Reconstruction as a Physical Theory Object* [23], and Paper III, *Empirical Realization, Observational Equivalence, and the Limits of Finite-Dimensional Classical Models* [24]. Twelve labels are kept strictly distinct: Definition, Axiom, Assumption, Conjecture, Lemma, Proposition, Theorem, Corollary, Interpretation, Empirical statement, Established physical result, New result claimed by this work. Three rules govern them, verbatim from the pilot contract: (i) an assumption is never converted into a theorem; (ii) every substantial claim is proved, cited, empirical, or explicitly conjectural; (iii) mathematical equivalence is never identified with physical equivalence without an empirical argument.[1]

[1] *Hedges, retained rather than tidied away.* Residually *uncertain*, flagged not asserted: the final page of Meyer 1973 (272 vs. 273). The journal data for Weinstein's 2009 survey was subsequently

2 Systems, states, observables

Definition 2.1 (System, observables, states)**.** A *finite-dimensional classical system* is a triple (M, ω, H): M smooth, second-countable, $\dim M = 2n$; $\omega \in \Omega^2(M)$ closed and nondegenerate; $H \in C^\infty(M)$; write $M^- := (M, -\omega)$. An *observable* is $f \in \mathcal{O} := C^\infty(M, \mathbb{R})$; with $\iota_{X_f}\omega = df$ and $\{f, g\} := \omega(X_f, X_g)$, $(\mathcal{O}, \{\cdot,\cdot\})$ is a Poisson algebra [2]. A *point state* is $x \in M$; a *statistical state* is a Borel probability measure, the point states being its Dirac measures. Unless stated otherwise M is connected and X_H complete — neither automatic, since a particle escaping to infinity in finite time violates completeness.

Axiom 2.2 (Hamiltonian kinematics and dynamics)**.** The state space of an isolated finite-dimensional classical system is a symplectic manifold, its observables the smooth real functions on it, and its evolution the flow Φ^t_H of X_H, so $\dot{f} = \{f, H\}$ [1, 2]. Labelled an *axiom*, not a theorem: nothing here derives it, and Section 6 exhibits ordinary mechanical systems violating it.

Assumption 2.3 (Observables)**.** Every $f \in C^\infty(M)$ is a physical observable.

Assumption 2.4 (Point states)**.** The physically realized states are the point states.

Both are false about laboratory practice (Empirical statement 5.6); both remain assumptions here, and are never converted into theorems.

3 Processes; partial composition

Definition 3.1 (Processes and the two categories)**.** Four strata: (a) the flow Φ^t_H; (b) a *symplectomorphism* $\psi : M_1 \to M_2$, $\psi^*\omega_2 = \omega_1$; (c) a *canonical relation* $L : M_1 \rightsquigarrow M_2$, a closed Lagrangian submanifold $L \subset M_1^- \times M_2$, so $\dim L = n_1 + n_2$ and $(-\omega_1 \boxplus \omega_2)|_L = 0$ [19]; (d) a *Dirac relation*, dropping nondegeneracy [6]. $\mathsf{FinSymp}$ has symplectic manifolds as objects and symplectomorphisms as morphisms; $\mathsf{SympRel}$ the same objects with canonical relations as candidate morphisms, composed by $L_2 \circ L_1 = \pi_{13}(F)$, $F := (L_1 \times L_2) \cap (M_1 \times \Delta_{M_2} \times M_3)$ — a canonical relation only when F is a clean intersection and $\pi_{13}|_F$ is proper and injective.

Established physical result 3.2 ($\mathsf{SympRel}$ is not a category)**.** Weinstein states the failure himself: canonical relations "compose well when a transversality condition is satisfied, but the failure of the most general compositions to be smooth manifolds means that the canonical relations do not comprise the morphisms of a category" [20].

Counterexample 3.3 (Composition that is not a submanifold)**.** Take $M_1 = \mathbf{1}$ (the one-point symplectic manifold), $M_2 = \mathbb{R}^4$ with $\omega = dq_1 \wedge dp_1 + dq_2 \wedge dp_2$, $M_3 = \mathbb{R}^2(q_1, p_1)$. Let $S(q_1, q_2) = \frac{1}{4}q_2^4 - \frac{1}{2}q_1 q_2^2$ and $L_1 := \operatorname{graph}(dS) = \{(q_1, q_2, -\frac{1}{2}q_2^2,\ q_2^3 - q_1 q_2)\}$, a closed embedded Lagrangian submanifold of dimension $2 = 0 + 2$. Let $W = \{p_2 = 0\}$, a hypersurface, hence coisotropic, with null distribution spanned by $X_{p_2} = \partial/\partial q_2$, and $R : M_2 \rightsquigarrow M_3$ its reduction relation $\{((q_1, q_2, p_1, 0), (q_1, p_1))\}$, of dimension $3 = 2 + 1$, with $-\omega_2 \boxplus \omega_3$ pulling back to $-dq_1 \wedge dp_1 + dq_1 \wedge dp_1 = 0$. Then $L_1 \cap W = \{q_2(q_2^2 - q_1) = 0\}$ is the union of the line $\{q_2 = 0\}$ and the parabola $\{q_1 = q_2^2\}$, meeting transversally at the origin — *not* a

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confirmed — it appeared as Port. Math. **67** (2010), 261–278, the form cited in Papers III and IV of this pilot — but the arXiv preprint is retained here as the citation of record for the version consulted.

submanifold, so the intersection is not clean. Since $\partial S/\partial q_1 = -\frac{1}{2}q_2^2$,

$$R \circ L_1 = \{(q_1, 0) : q_1 \in \mathbb{R}\} \ \cup \ \{(q_1, -\tfrac{1}{2}q_1) : q_1 \geq 0\},$$

a line with a half-ray attached at the origin: not a one-dimensional submanifold of $\mathbb{R}^2$, so $R \circ L_1$ is not a canonical relation. □

4 Composition I: monoidal, not cartesian

The *direct product* of (M_1, ω_1, H_1) and (M_2, ω_2, H_2) is $(M_1 \times M_2,\ \mathrm{pr}_1^*\omega_1 + \mathrm{pr}_2^*\omega_2,\ H_1 \circ \mathrm{pr}_1 + H_2 \circ \mathrm{pr}_2)$, with unit $\mathbf{1}$.

Proposition 4.1 (No projections)**.** *Let* $\dim M = 2n$, $\dim N = 2m$, $m \geq 1$. *The graph* $\Gamma = \{((x, y), x)\} \subset (M \times N)^- \times M$ *of* pr_M *is* not *a canonical relation.*

Proof. Lagrangian submanifolds of $(M \times N)^- \times M$ have dimension $\big((2n + 2m) + 2n\big)/2 = 2n+m$, while $\dim \Gamma = 2n+2m$; these agree only if $m = 0$. Independently, the pullback of $-(\omega_M \boxplus \omega_N) \boxplus \omega_M$ along $(x, y) \mapsto ((x, y), x)$ is $-\omega_M - \omega_N + \omega_M = -\omega_N \neq 0$, so Γ is not even isotropic. □

Theorem 4.2 (No diagonal)**.** *Let* $\dim M = 2n \geq 2$. *The graph* $\Gamma_\delta = \{(x, (x, x)) : x \in M\}$ *of the set-theoretic diagonal is not a canonical relation* $M \rightsquigarrow M \times M$*.*

Proof. $M^- \times (M \times M)$ has dimension $6n$, so its Lagrangian submanifolds have dimension $3n$, whereas $\dim \Gamma_\delta = 2n$, and $3n \neq 2n$ for $n \geq 1$. The stronger, convention-independent obstruction is isotropy: the pullback of $(-\omega) \boxplus \omega \boxplus \omega$ along $x \mapsto (x, x, x)$ is $-\omega + \omega + \omega = \omega$, nondegenerate. So Γ_δ is a *symplectic* submanifold — maximally far from isotropic. □

Corollary 4.3 (Not cartesian)**.** *No* copy relation *— a canonical relation graphing* $x \mapsto (x, x)$ *— exists,* Γ_δ *being the only candidate. By the characterization of cartesian structure through natural diagonals and projections (Fox's theorem [12]; for the monoidal background see [13]),* $(\mathsf{SympRel}, \times, \mathbf{1})$ *is symmetric monoidal but not cartesian. This is* not *a classical no-cloning theorem: statistical states can be copied by preparing two systems identically, and the obstruction is internal to a chosen morphism class. Reading it as a physical impossibility would be the mathematical-for-physical substitution the contract forbids.*

5 Composition II: decomposition is not intrinsic

Lemma 5.1 (Non-interaction equals product flow)**.** *For a symplectic splitting* $M \cong M_1 \times M_2$ *with* M_1, M_2 *connected,* $H = H_1 \circ \mathrm{pr}_1 + H_2 \circ \mathrm{pr}_2$ *up to a constant* iff $X_H = (X_{H_1}, X_{H_2})$.

Proof. Write $X_H = (V(x), W(y))$. Then $dH = \iota_V\omega_1 + \iota_W\omega_2$ splits by bidegree, so $\partial^2 H/\partial x\, \partial y = 0$ and $H(x, y) = H(x, y_0) + H(x_0, y) - H(x_0, y_0)$; the converse is the same computation backwards. □

New result claimed by this work 5.2 (Interaction is not an invariant of (M, ω, H))**.** Let $M = \mathbb{R}^4$ with $\omega = dq_1 \wedge dp_1 + dq_2 \wedge dp_2$ and

$$H = \frac{p_1^2 + p_2^2}{2m} + \frac{k}{2}(q_1^2 + q_2^2) + \frac{c}{2}(q_1 - q_2)^2, \qquad m > 0,\ k > 0,\ c \neq 0,\ k + 2c > 0.$$

Then (M, ω) admits two symplectic direct-product decompositions

$$\mathbf{A}:\ M \cong \mathbb{R}^2(q_1, p_1) \times \mathbb{R}^2(q_2, p_2), \qquad \mathbf{B}:\ M \cong \mathbb{R}^2(Q_+, P_+) \times \mathbb{R}^2(Q_-, P_-),$$

$Q_\pm = (q_1 \pm q_2)/\sqrt{2}$, $P_\pm = (p_1 \pm p_2)/\sqrt{2}$, such that H is interacting with respect to $\mathbf{A}$ and non-interacting with respect to $\mathbf{B}$. Hence "is composite" and "its parts interact" are properties not of (M, ω, H) but of the pair $\big((M, \omega, H), \text{decomposition}\big)$ — of the system with a choice of which mutually commutant Poisson subalgebras count as local. *Priority hedge:* the computation is elementary and we expect it is folklore; our audit located no published statement of the *classical* case, only the quantum tensor-product results of Zanardi and of Zanardi, Lidar and Lloyd [21, 22], outside scope and a boundary test only. We claim the result while disclaiming novelty of the computation.

*Proof. **B** is symplectic.* $dQ_+ \wedge dP_+ + dQ_- \wedge dP_- = \frac{1}{2}[(dq_1 + dq_2) \wedge (dp_1 + dp_2) + (dq_1 - dq_2) \wedge (dp_1 - dp_2)] = \omega$. In matrix form the transformation is $T = \operatorname{diag}(S, S)$ on (q, p) with $S = 2^{-1/2}\left(\begin{smallmatrix} 1 & 1 \\ 1 & -1 \end{smallmatrix}\right) \in O(2)$, and $T^\mathsf{T} J T = J$ because $S^\mathsf{T} S = I$; so $T \in Sp(4, \mathbb{R})$.

*H decouples in **B**.* Orthogonality of S gives $p_1^2 + p_2^2 = P_+^2 + P_-^2$, $q_1^2 + q_2^2 = Q_+^2 + Q_-^2$, $q_1 - q_2 = \sqrt{2}\, Q_-$, so $H = \big[P_+^2/2m + \frac{k}{2} Q_+^2\big] + \big[P_-^2/2m + \frac{k+2c}{2} Q_-^2\big]$, uncoupled oscillators of frequencies $\sqrt{k/m}$, $\sqrt{(k + 2c)/m}$.

*H does not decouple in **A**.* If $H = a(q_1, p_1) + b(q_2, p_2)$ then $\partial^2 H / \partial q_1 \partial q_2 = 0$; but $\partial H / \partial q_1 = k q_1 + c(q_1 - q_2)$, so $\partial^2 H / \partial q_1 \partial q_2 = -c \neq 0$. □

The ambiguity is large: $Sp(4, \mathbb{R})$ acts transitively on ordered linear symplectic splittings of $\mathbb{R}^4$ with stabilizer $Sp(2, \mathbb{R}) \times Sp(2, \mathbb{R})$, a 4-parameter family already.

Conjecture 5.3 (Generic non-decomposability)**.** For generic H on a compact symplectic manifold of dimension ≥ 4, no global symplectic product decomposition makes H non-interacting. We have no proof: by Lemma 5.1 it would force an invariant foliation of codimension n, which KAM-type genericity should obstruct. *Conjecture, not theorem.*

Definition 5.4 (Compositional system)**.** A *compositional system* is $(M, \omega, H, \mathcal{A}_1, \mathcal{A}_2)$ with $\mathcal{A}_1, \mathcal{A}_2 \subset C^\infty(M)$ Poisson subalgebras that are each other's commutants and generate a dense subalgebra; equivalently an integrable symplectic-orthogonal splitting $TM = V_1 \oplus V_2$. Write $\mathsf{CompSys}$ for the category, HamSys for that of systems (M, ω, H) with H-preserving symplectomorphisms, $U : \mathsf{CompSys} \to \mathsf{HamSys}$ for the forgetful functor.

Corollary 5.5. *U is not injective on isomorphism classes over a fixed (M, ω, H): Theorem 5.2 gives two objects with the same image disagreeing on the interaction predicate. Compositional structure is* strictly additional data.

Empirical statement 5.6 (Units, finite resolution, correlations)**.** $q + p$ is smooth and dimensionally meaningless; every reading has finite precision $\varepsilon > 0$; on a constraint surface only gauge-invariant functions are measured; the decomposition is fixed in practice by spatial separation. So Assumption 2.3 is false about laboratory observables, and locality of apparatus is an empirical input, not a consequence of Axiom 2.2. Assumption 2.4 fails too:

a joint measure on $M_1 \times M_2$ is *not* determined by its marginals, and for chaotic systems a point state is not realizable at precision ε over a horizon T.

New result claimed by this work 5.7 (Status of the founding thesis in this scope)**.** The strong reading of the pilot thesis — that compositional structure is determined by the mathematics of a physical system — is *false* here, by Theorem 5.2 and Corollary 5.5. Only the weak reading survives, "empirically realized" carrying the load: the decomposition datum is what a measurement arrangement supplies (reading q_1 needs an apparatus coupled to particle 1 alone, Q_+ one coupled to both). That is an interpretive correspondence, not a theorem, and it can fail — an apparatus may couple to a non-integrable family of observables, for which no $\mathcal{A}_i$ exists.

6 Composition III: interconnection is not composition

Remark 6.1 (A correction to the usual framing)**.** It is often said that "composition fails to preserve Hamiltonian structure". That is mis-stated: under the cleanness and properness conditions of Definition 3.1, composition of canonical relations succeeds. What fails is a family of *different* operations also called composition — restriction to a nonintegrable distribution, addition of a non-Hamiltonian field, restriction to a non-constant-rank constraint set, quotient by a non-free action — items (a)–(d).

Lemma 6.2 (Trace and spectrum obstructions)**.** *(i) If $X = X_H$ for some symplectic form near an equilibrium p, then in Darboux coordinates $DX(p) = \begin{pmatrix} H_{qp} & H_{pp} \\ -H_{qq} & -H_{pq} \end{pmatrix}$, so* $\operatorname{tr} DX(p) = 0$*, a diffeomorphism invariant. (ii) A linear Hamiltonian field is $A = JS$, $S = S^{\mathsf{T}}$, so* $\operatorname{spec}(A) = -\operatorname{spec}(A)$*. Either violation certifies non-Hamiltonicity for* every *symplectic form near p.*

Established physical result 6.3 ((a) Nonholonomic: almost-Poisson brackets)**.** For a constraint distribution $D \subset TQ$, d'Alembert's principle induces an *almost-Poisson* bracket on the constrained phase space whose Jacobiator vanishes identically *iff* D is integrable, i.e. iff the constraints are holonomic [18, 10]; Koon and Marsden state it as failure of the reduced two-form to be closed. The vertical rolling disk is the standard witness, its distribution being bracket-generating.

Proposition 6.4 (Chaplygin sleigh: energy conserved, dynamics not Hamiltonian)**.** *The reduced sleigh equations on $\mathbb{R}^2(v, \varpi)$ are $\dot{v} = a\varpi^2$, $\dot{\varpi} = -\nu v \varpi$, $\nu = ma/(I + ma^2) > 0$, with $E = \frac{1}{2}mv^2 + \frac{1}{2}(I + ma^2)\varpi^2$ conserved: $\dot{E} = mav\varpi^2 - (I + ma^2)\nu v \varpi^2 = 0$. Yet at each equilibrium $(v_0, 0)$, $v_0 \neq 0$, the Jacobian $\left(\begin{smallmatrix} 0 & 0 \\ 0 & -\nu v_0 \end{smallmatrix}\right)$ has trace $-\nu v_0 \neq 0$, so by Lemma 6.2(i) no symplectic structure near $(v_0, 0)$ makes the flow Hamiltonian. On $\{\pm\varpi > 0\}$ the flow* is *Hamiltonian for E with respect to $\Omega = \big((I + ma^2)/(a\varpi)\big) dv \wedge d\varpi$, which blows up across $\{\varpi = 0\}$ and admits no smooth extension.*

Proposition 6.5 ((b) Dissipative coupling leaves the class)**.** *Couple two identical oscillators by a relative-velocity damper: $\dot{q}_i = p_i/m$, $\dot{p}_1 = -kq_1 - (\gamma/m)(p_1 - p_2)$, $\dot{p}_2 = -kq_2 - (\gamma/m)(p_2 - p_1)$, $\gamma > 0$. In the normal-mode coordinates of Theorem 5.2 the $+$ mode is undamped with spectrum $\pm i\sqrt{k/m}$, while the $-$ mode obeys $\lambda^2 + (2\gamma/m)\lambda + k/m = 0$, both roots in the open left half-plane. The spectrum is not closed under $\lambda \mapsto -\lambda$ and the trace at the origin is $-2\gamma/m \neq 0$; by Lemma 6.2 the interconnection is not Hamiltonian near the origin, although both factors are.*

Counterexample 6.6 ((c) Non-constant rank: no smooth reduced space)**.** In $\mathbb{R}^4$ with $\omega = dq_1 \wedge dp_1 + dq_2 \wedge dp_2$ let $C = \{p_1 = 0,\ p_2 = q_1^2/2\}$, a surface parametrized by $(u, v) \mapsto (u, v, 0, u^2/2)$. Then $\iota^*\omega = dv \wedge (u\,du) = -u\,du \wedge dv$, of rank 2 on $\{q_1 \neq 0\}$ and 0 on $\{q_1 = 0\}$. The characteristic distribution is not a distribution, so $C/\ker$ carries no smooth manifold structure and the Gotay–Nester–Hinds algorithm [9] does not terminate in a reduced symplectic manifold. Equivalently, with $\Phi = (p_1,\ p_2 - q_1^2/2)$ one computes $\{\Phi_1, \Phi_2\} = q_1$, so the Dirac matrix has determinant q_1^2: the constraints are second class off $\{q_1 = 0\}$ and first class on it. The *class* of the constraints is not constant, which is exactly what breaks the Dirac bracket. □

Counterexample 6.7 ((d) Singular reduction leaves the manifolds)**.** Reduction of a Hamiltonian G-space at a regular value of the momentum map, with the G_μ-action free and proper, yields a symplectic manifold $J^{-1}(\mu)/G_\mu$ [14, 15]; both hypotheses are essential. Let $M = \mathbb{C}^2$ with its standard form and let S^1 act with weights $(1, -1)$, the 1:-1 resonance. The momentum map is $J = \frac{1}{2}(|z_1|^2 - |z_2|^2)$; the action is free off the origin and has full isotropy at 0, so $\mu = 0$ is the unique singular value. The invariant ring is generated by $\sigma_1 = |z_1|^2$, $\sigma_2 = |z_2|^2$, $\sigma_3 = \mathrm{Re}(z_1 z_2)$, $\sigma_4 = \mathrm{Im}(z_1 z_2)$ subject to $\sigma_3^2 + \sigma_4^2 = \sigma_1\sigma_2$, and on $J^{-1}(0)$ we have $\sigma_1 = \sigma_2 =: s \geq 0$, whence

$$M_0 = J^{-1}(0)/S^1 \ \cong\ \{(s, \sigma_3, \sigma_4) \in \mathbb{R}^3 :\ \sigma_3^2 + \sigma_4^2 = s^2,\ s \geq 0\},$$

a cone, singular at the apex: homeomorphic to $\mathbb{R}^2$ but not a smooth symplectic manifold. It is a symplectic *stratified* space in the sense of Sjamaar and Lerman [16]. □

Established physical result 6.8 (The compositionally closed class is Dirac, not symplectic)**.** The composition of two Dirac structures is again a Dirac structure [6, 5], so port-Hamiltonian systems built on them are closed under power-conserving interconnection [17]. Symplectic and Poisson structures lack that closure, as (a)–(d) show; closure requires enlarging the objects (to Dirac / port-Hamiltonian systems, or for dissipation contact systems [3]) or accepting a partial composition. Cervera, van der Schaft and Baños note that even Dirac composition can fail in infinite dimensions.

7 Empirical realization, and a no-go

Established physical result 7.1 (No-interaction theorem: Currie–Jordan–Sudarshan; Leutwyler)**.** Assume: (H1) finitely many point particles, $N \geq 2$, on a finite-dimensional phase space with the canonical Poisson bracket; (H2) a canonical realization of the Poincaré algebra by phase-space functions, H generating time evolution (Dirac's instant form [8]); (H3) the particle positions at a common time are half of a canonical system of variables, transforming under the Euclidean subgroup canonically and geometrically in the same way; (H4) the world-line condition, that the set of world lines is frame-independent; (H5) nondegeneracy. Then the only compatible evolution is *free*: all accelerations vanish. Proved for $N = 2$ by Currie, Jordan and Sudarshan [7], for arbitrary N by Leutwyler [11].

Interpretation 7.2. There is no compositional theory of *interacting relativistic subsystems* in this scope. Each escape damages a different part of the picture: dropping finite-dimensionality imports field degrees of freedom; dropping canonical positions means subsystem identity is no longer read off the phase space; dropping the world-line condition makes "which subsystem is where" frame-dependent. This interprets a theorem; it does not extend one.

Hidden assumptions, made explicit and separately falsifiable

Assumption 7.3 (Monoidal composition). $A \otimes B$ is determined by A and B up to canonical isomorphism. *Status: refuted as stated.* For $H = H_1 + H_2 + V(q_1, q_2)$ the factors do not determine H — two masses joined by springs of stiffness $k \neq k'$ are different composites of the same parts — so no bifunctor on pairs (phase space, Hamiltonian) produces interacting composites: the interaction term is irreducibly extra data. Independently, for pair potentials decaying as $r^{-\alpha}$, $\alpha \leq d$, energy is not additive at all, which underlies ensemble inequivalence [4] and covers gravitational and unscreened Coulomb systems.

Assumption 7.4 (Intrinsic subsystems). A composite determines its decomposition into parts. *Status: refuted* by Theorem 5.2 and Corollary 5.5. Adversarially: ask the framework how many particles (M, ω, H) has. It cannot say; particle number is not a symplectic invariant.

Assumption 7.5 (Isomorphism implies physical equivalence). Symplectomorphic systems with conjugate Hamiltonians are physically equivalent. *Status: refuted, and forbidden by the pilot contract.* A pendulum near equilibrium, a mass–spring and an LC circuit are conjugate at leading order but differ in units, measurement procedure, domain of validity and failure mode (separatrix, yield point, breakdown voltage); and since H is defined only up to a constant and a symplectomorphism, "energy" is not an invariant of the isomorphism class.

Assumption 7.6 (Closure under interconnection). The finite-dimensional symplectic Hamiltonian systems are closed under composition. *Status: refuted four independent ways* — Established result 6.3 with Proposition 6.4; Proposition 6.5; Counterexample 6.6; Counterexample 6.7 — the class-level repair being Established result 6.8.

Assumption 7.7 (Compositional realization). The interpretation map commutes with composition. *Status: refuted for long-range forces* (non-additivity, Assumption 7.3); an unexamined isolation/screening assumption otherwise. This is what the founding thesis most effectively conceals: "empirically realized compositional structure" presumes realization respects composition — a claim about interaction ranges, not a mathematical fact.

Assumption 7.8 (Autonomy). A realized system is an autonomous Hamiltonian system on a symplectic manifold. *Status: refuted* by Liouville's theorem [2, §16]: any system with a volume-contracting attractor — every real mechanical system with friction — is not one. Precision matters: a damped oscillator does admit a time-dependent Lagrangian of Caldirola–Kanai type, so "not Hamiltonian" is false while "not autonomous symplectic, and its Hamiltonian is not the energy" is true.

8 Unresolved tensions, and conclusion

U1, totality versus expressiveness: the direct product is total but expresses only *non*-interacting composites, while interconnection is expressive but partial. No finite-dimensional construction known to us is both, and Established result 3.2 records that SympRel, the attempt, fails. **U2, reduction destroys compositeness:** the two-body Kepler problem, reduced by translations — mandatory, absolute position not being observable — becomes a one-body problem. A composite reduces to a non-composite; we regard this as the sharpest single tension. **U3, exact unit, approximate isolation:** $A \otimes \mathbf{1} \cong A$ holds on the nose, whereas physical isolation is an approximation that fails outright for long-range forces (Assumption 7.7). We know of no monoidal category coherent only up to ε.

The thesis is neither confirmed nor idle here; it is *corrected*. Physics, in this scope, is the study of mathematical structures *together with* an empirically supplied compositional datum that the mathematics does not determine.

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