Hamiltonian Reconstruction as a Physical Theory Object

A typed reconstruction of finite-dimensional mechanics, with its non-examples, limits and falsification attempts

PRINCIPIA PHYSICA Pilot — Topic 2

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Abstract

We reconstruct finite-dimensional Hamiltonian mechanics as a *typed physical theory object*, separating bare structure from an interpretation layer, and prove the results that make its slots well typed (nondegeneracy $\Leftrightarrow$ uniqueness of X_H; Jacobi $\Leftrightarrow$ $d\omega = 0$; conservation; Liouville; Noether). Three findings are structural: dynamical composition is *rigid*, so "system $\mapsto$ dynamics" is no monoidal functor on the Cartesian product; interconnection is not morphism composition and its Lagrangian-relation replacement is partial, with four failure modes; and isomorphism underdetermines physical identity. Non-examples, limits and falsification cases follow, each typed as proved, cited, empirical, conjectural, or unverified.

1 Introduction and epistemic conventions

Can finite-dimensional Hamiltonian mechanics be written as an empirically interpreted compositional theory object *without conflating mathematical with physical equivalence*? Scope is finite-dimensional smooth classical mechanics; field theory, general relativity and quantum theory enter only as boundary tests (Sec. 8). Three rules bind: an assumption is never promoted to a theorem; every substantial claim is proved, cited, empirical, or conjectural; and mathematical equivalence is never identified with physical equivalence without empirical argument. Twelve epistemic labels share one counter, so every claim shows its type. Companion papers are Paper I [10] and Paper III [11].

Assumption 1.1 (Novelty caveat; granted, not argued)**.** Items labelled *New result* are new only as typed formulations inside this reconstruction; the mathematics is classical and no priority search was made.

Conventions. $\iota_{X_f}\omega = df$, $\{f, g\} := \omega(X_f, X_g)$, Φ^t_H the (possibly local) flow of X_H, frequencies ν. The exterior derivative uses the determinant convention,

$$\begin{aligned} d\omega(X, Y, Z) = {} & X\omega(Y, Z) - Y\omega(X, Z) + Z\omega(X, Y) \\ & - \omega([X, Y], Z) + \omega([X, Z], Y) \\ & - \omega([Y, Z], X). \end{aligned} \tag{1}$$

2 The typed signature

Definition 2.1 (Bare Hamiltonian theory object)**.** $\mathcal{H} = (M, \omega, \mathcal{O}, \flat, \sharp, H, X_H, \Phi_H)$, no hypothesis tacit. M: smooth, finite-dimensional, Hausdorff, second countable (hence partitions of unity), boundaryless, connected only where flagged. ω: closed (Ax. 2.2), nondegenerate (Ax. 2.3), forcing $\dim M = 2n$ (Prop. 6.1). $\mathcal{O} = C^\infty(M, \mathbb{R})$, with $\{f, g\}$ Poisson iff $d\omega = 0$ (Cor. 3.4); $\omega^\flat : X \mapsto \iota_X\omega$ and $\omega^\sharp$ are isomorphisms iff ω is nondegenerate (Prop. 3.1). $H \in \mathcal{O}$: time-independent, $X_H = \omega^\sharp dH$ unique and smooth but not necessarily complete (Assum. 2.4). Φ_H: local flow on open D, a one-parameter group in $\mathrm{Symp}(M, \omega)$ only if X_H is complete.

Axiom 2.2 (Closedness)**.** $d\omega = 0$.

Axiom 2.3 (Nondegeneracy)**.** $\omega^\flat_m : T_mM \to T^*_mM$ is injective for every m.

These are axioms *of the object*, not theorems about nature; they fail for real systems in identifiable ways (Secs. 6–8).

Assumption 2.4 (Completeness; granted where used)**.** Where a statement quantifies over all $t \in \mathbb{R}$ we assume X_H complete—*false* for Kepler on $T^*(\mathbb{R}^3 \setminus \{0\})$ and for $H = p^2/2 - q^4$ (Prop. 7.1), and never a theorem.

Definition 2.5 (Processes, composition, interpretation)**.** Composition is $\mathcal{H}_1 \boxtimes \mathcal{H}_2 := (M_1 \times M_2,\ \pi_1^*\omega_1 + \pi_2^*\omega_2,\ H_1 \circ \pi_1 + H_2 \circ \pi_2)$, unit $\mathbf{1}$. The *interpreted* object is $\widehat{\mathcal{H}} = (\mathcal{H}; P, \iota, u, \tau, \mathfrak{p})$: measurement procedures P (apparatus specifications, not mathematical objects), $\iota : P \to \mathcal{O}$ fixing which observable each reads, dimensions u, clock calibration τ, preparations $\mathfrak{p}$. What can be said with $\mathcal{H}$ alone is mathematics; anything physical needs $\widehat{\mathcal{H}}$.

Let $\mathsf{FinSymp}$ have objects (M, ω) and morphisms symplectomorphisms, HamSys objects (M, ω, H) with $\varphi^*\omega' = \omega$, $H' \circ \varphi = H$. Every such morphism is a diffeomorphism, so both are *groupoids*: interconnection cannot be morphism composition.

3 Core reconstruction

The apparatus is standard [1, 2]; each proof shows the hypothesis it consumes.

Proposition 3.1 (Nondegeneracy and the dynamics slot)**.** *The following are equivalent: (1) ω is nondegenerate; (2) $\omega^\flat : TM \to T^*M$ is a bundle isomorphism; (3) $\Gamma(\omega^\flat) : \mathfrak{X}(M) \to \Omega^1(M)$ is a $C^\infty(M)$-module isomorphism. Then each H determines a unique smooth X_H with $\iota_{X_H}\omega = dH$.*

Proof. $\omega^\flat$ is C^∞-linear since $\iota_{fX}\omega = f\iota_X\omega$, giving (2)⇒(3) with inverse $\Gamma(\omega^\sharp)$. (1)⇒(2): $\omega^\flat_m$ is injective between equidimensional spaces, hence bijective, and (ω_{ij}) is smooth with nowhere-vanishing determinant, so $\omega^\sharp$ is smooth by Cramer's rule. (3)⇒(1): if $\omega^\flat_m$ is not injective it is not surjective, so pick $\xi \notin \operatorname{im}\omega^\flat_m$ and cut it off with a bump function (*here second countability is load-bearing*) to get α with $\alpha(m) = \xi$; surjectivity gives $\iota_X\omega = \alpha$, a contradiction. Then $X_H := \Gamma(\omega^\sharp)(dH)$ is the unique preimage of dH. □

Proposition 3.2 (Bracket identities; closedness not yet needed)**.** $\{f, g\} = df(X_g) = X_gf = -X_fg$*, and* $\{\cdot, \cdot\}$ *is* $\mathbb{R}$*-bilinear, antisymmetric and Leibniz.*

Proof. $\omega(X_f, X_g) = (\iota_{X_f}\omega)(X_g) = df(X_g)$; bilinearity from $\mathbb{R}$-linearity of d, antisymmetry from ω, and $\{fg, h\} = d(fg)(X_h) = f\{g, h\} + g\{f, h\}$. $\square$

Theorem 3.3 (Jacobiator equals $-d\omega$ on Hamiltonian fields). *$d\omega(X_f, X_g, X_h) = -J(f, g, h)$, where $J(f, g, h) := \{\{f, g\}, h\} + \{\{g, h\}, f\} + \{\{h, f\}, g\}$.*

Proof. From $\iota_{X_h}\omega = dh$ and $X_u v = \{v, u\}$,

$$\omega([X_f, X_g], X_h) = -\{\{h, g\}, f\} + \{\{h, f\}, g\},$$

and cyclically for the other two commutators, while $X_f\omega(X_g, X_h) = \{\{g, h\}, f\}$ and cyclic. Substituting into (1), the coefficients of $\{\{g, h\}, f\}$, $\{\{f, h\}, g\}$, $\{\{f, g\}, h\}$ are $+1-1-1 = -1$, $-1+1+1 = +1$, $+1-1-1 = -1$; since $\{\{f, h\}, g\} = -\{\{h, f\}, g\}$, the total is $-J(f, g, h)$. (The sign is derived in full above; it was additionally cross-checked by exact rational arithmetic during drafting, Sec. 9.) $\square$

Corollary 3.4 (The observable slot is Poisson iff the form is closed). *$(\mathcal{O}, \cdot, \{\cdot, \cdot\})$ is a Poisson algebra $\iff$ $d\omega = 0$; then $[X_f, X_g] = X_{\{f,g\}}$ and $f \mapsto X_f$ is a Lie algebra homomorphism with kernel the locally constant functions.*

Proof. One direction is Thm. 3.3. Conversely, $d\omega$ is a tensor, so $d\omega_m$ depends only on the values $X_f(m)$, $X_g(m)$, $X_h(m)$. By Prop. 3.1 every covector at m is some $df(m)$, so these exhaust T_mM and $J \equiv 0$ forces $d\omega_m = 0$. $\square$

Proposition 3.5 (Conservation). *$\frac{d}{dt}(H \circ \Phi_H^t) = 0$, and f is constant along every integral curve of X_H iff $\{f, H\} = 0$.*

Proof. $dH(X_H) = \omega(X_H, X_H) = 0$ by antisymmetry alone—closedness is not used—and $\frac{d}{dt}(f \circ \Phi_H^t)(m) = \{f, H\}(\Phi_H^t m)$; evaluate at $t = 0$ for the converse. $\square$

Theorem 3.6 (Liouville invariance). *$(\Phi_H^t)^*\omega = \omega$ on the domain of the (possibly local) flow, hence $(\Phi_H^t)^*\Lambda = \Lambda$ for $\Lambda = \omega^n/n!$.*

Proof. $\mathcal{L}_{X_H}\omega = \iota_{X_H}d\omega + d\iota_{X_H}\omega = 0 + d(dH) = 0$ by Cartan, so $\frac{d}{dt}(\Phi_H^t)^*\omega = 0$ with $\Phi_H^0 = \mathrm{id}$, and pullback is an algebra map. Both axioms are used: Ax. 2.2 kills the first term, exactness of $\iota_{X_H}\omega$ the second. $\square$

Theorem 3.7 (Noether, finite-dimensional Hamiltonian form). *Let a symplectic G-action have generators ξ_M and admit a momentum map $J : M \to \mathfrak{g}^*$, i.e. $\iota_{\xi_M}\omega = dJ_\xi$ with $J_\xi = \langle J, \xi\rangle$. If H is G-invariant then $\{J_\xi, H\} = 0$ for all ξ.*

Proof. G-invariance gives $\xi_M[H] = 0$, so $\{J_\xi, H\} = \omega(X_{J_\xi}, X_H) = \omega(\xi_M, X_H) = -dH(\xi_M) = 0$. $\square$

Remark 3.8 (The momentum map need not exist). J_ξ exists globally only if $[\iota_{\xi_M}\omega] = 0$ in $H^1_{\mathrm{dR}}(M)$: on $\mathbb{R}^2/\mathbb{Z}^2$, $X = \partial_x$ gives $\iota_X\omega = dy$, closed but not exact, so X preserves ω yet has *no global energy function*—"every symmetry yields a conserved quantity" is false as stated.

4 Composition, and where it breaks

On $M_1 \times M_2$ with $\omega = \pi_1^*\omega_1 + \pi_2^*\omega_2$ the form is symplectic and $\{\pi_1^* f, \pi_2^* g\} = 0$ identically.

New result claimed by this work 4.1 (Interaction rigidity)**.** Let $H = H_1 \circ \pi_1 + H_2 \circ \pi_2 + V$ with $V \in C^\infty(M_1 \times M_2)$ arbitrary; for $V = 0$, Prop. 3.1 gives $X_H = (X_{H_1}, X_{H_2})$, integrating componentwise. Then: (i) $X_H = (X_{H_1}, X_{H_2}) + X_V$ is Hamiltonian, and Thm. 3.6 and total-energy conservation hold verbatim; (ii) *kinematic composition is untouched*: ω is still the direct sum and the factor subalgebras still commute *regardless of* V; (iii) *dynamical composition is rigid*: for M_1 connected the following are equivalent—(a) $\pi_1 \circ \Phi_H^t = \Phi_{H_1}^t \circ \pi_1$ for all t; (b) $T\pi_1 \circ X_H = X_{H_1} \circ \pi_1$; (c) $\{\pi_1^* f, V\} = 0$ for all $f \in C^\infty(M_1)$; (d) $V = \pi_2^* k$, i.e. *there is no interaction at all*; (iv) $H_i \circ \pi_i$ is conserved iff $\{\pi_i^* H_i, V\} = 0$; (v) coupling data form the affine space $C^\infty(M_1 \times M_2)/(\pi_1^* C^\infty(M_1) + \pi_2^* C^\infty(M_2))$, so *interconnection is extra data, not determined by the parts*; (vi) hence $(M, \omega) \mapsto (M, \omega, H)$ is *not* a monoidal functor $\mathsf{FinSymp} \to \mathsf{HamSys}$ under interaction, and Cartesian composition is compositional *kinematically only.*

Proof of (iii). (a)$\Leftrightarrow$(b) is the φ-relatedness criterion; subtracting the split field, (b)$\Leftrightarrow$ $T\pi_1 \circ X_V = 0 \Leftrightarrow X_V(\pi_1^* f) = 0$ for all f, which is (c). (c)$\Rightarrow$(d): in a Darboux chart on M_1, $f = q^i$ and $f = p_i$ give $\partial V/\partial p_i = \partial V/\partial q^i = 0$, so all M_1-partials vanish; M_1 is connected, so $V = \pi_2^* k$. (d)$\Rightarrow$(c) is immediate. □

Conjecture 4.2 (No total interconnection operation)**.** There is no everywhere-defined associative operation on interpreted finite-dimensional Hamiltonian theory objects that restricts to $\boxtimes$ on non-interacting pairs, is functorial for the induced dynamics, and implements interconnection by shared variables rather than by product. We have neither proof nor citation; Sec. 7 shows the natural candidate is only partially defined, which is evidence for, not proof of, it. The behavioural framework [16] gives a total operation on a *different* object type and so does not settle it.

5 Equivalence is not physical identity

Write $\mathcal{H}_1 \cong \mathcal{H}_2$ when a diffeomorphism φ has $\varphi^*\omega_2 = \omega_1$ and $H_2 \circ \varphi = H_1$, so $\varphi_* X_{H_1} = X_{H_2}$; weaker grades keep less, as $H_2 \circ \varphi = H_1 + c$ loses the zero of energy, $(c\,\omega_1, cH_1)$ the unit of action, and $\varphi_* X_{H_1} = \rho X_{H_2}$ all but the orbit foliation.

New result claimed by this work 5.1 (Mass is not an isomorphism invariant; frequency is)**.** On $T^*\mathbb{R}$ let $\varphi_\lambda(q,p) = (\lambda q, p/\lambda)$, $\lambda > 0$; then $\varphi_\lambda^*\omega = \omega$ and, with $H_{m,\nu} = \frac{p^2}{2m} + \frac{1}{2} m\nu^2 q^2$, $H_{m,\nu} \circ \varphi_\lambda = H_{\lambda^2 m,\nu}$: m and k scale by λ^2 while ν is fixed, so a 1 g and a 1 kg oscillator of equal frequency are isomorphic as bare objects. The class fixes ν, equivalently the action $I = E/\nu$, not m or k; mass is recovered only by fixing ι, u. Likewise the oscillator and the LC circuit $\frac{\Phi^2}{2L} + \frac{Q^2}{2C}$, $\{\Phi, Q\} = 1$, are strictly isomorphic under $m \leftrightarrow C$, $k \leftrightarrow 1/L$.

New result claimed by this work 5.2 (Physical identity is not a property of $\mathcal{H}$)**.** By 5.1, $\mathcal{H} \mapsto \widehat{\mathcal{H}}$ is not injective on isomorphism classes: physical identity is a property of $(\mathcal{H}, \iota, u, \tau, \mathfrak{p})$, never of $\mathcal{H}$ alone, so "symplectomorphism = physical equivalence" is unlicensed.

Lemma 5.3 (Twisted forms; exactness can fail)**.** *For closed σ, $\omega_{\mathrm{can}} + \pi^*\sigma$ is symplectic on T^*Q; for $\sigma = -eF$ it gives the Lorentz force with p kinetic, isomorphic to minimal* *coupling with p canonical iff $F = dA$ globally—which fails for the monopole on $\mathbb{R}^3 \setminus \{0\}$. The typings then differ only in ι, and nothing in (M, ω, H) adjudicates.*

6 Non-examples

Proposition 6.1 (Odd dimension)**.** *A manifold of odd dimension admits no nondegenerate 2-form, hence none symplectic.*

Proof. Nondegeneracy needs $A = (\omega_{ij}(m))$ invertible and antisymmetric, but then

$$\det A = (-1)^{\dim M} \det A = -\det A,$$

so $\det A = 0$. $\square$

Proposition 6.2 (Closed manifolds)**.** *No closed manifold with $H^2_{\mathrm{dR}}(M; \mathbb{R}) = 0$ is symplectic; in particular S^{2n} is symplectic iff $n = 1$, so S^4 is not. No closed symplectic manifold has an* exact *symplectic form.*

Proof. If ω is symplectic on closed M^{2n} then ω^n is a volume form, so $\int_M \omega^n \neq 0$, whence $[\omega] \neq 0$, contradicting $H^2_{\mathrm{dR}} = 0$; and $H^2_{\mathrm{dR}}(S^{2n}) = 0$ for $n \geq 2$. If $\omega = d\alpha$ then $\omega^n = d(\alpha \wedge \omega^{n-1})$ and $\int_M \omega^n = 0$ by Stokes. Compactness is essential: $\mathbb{R}^{2n}$ and T^*Q are exact and admissible. $\square$

Counterexample 6.3 (Damped oscillator: no invariant area form at all)**.** On $\mathbb{R}^2$, $X = \frac{p}{m}\partial_q - (kq + \gamma p)\partial_p$ with $\gamma, k > 0$ gives $d(\iota_X \omega) = -\gamma\omega \neq 0$, so $\iota_X \omega$ is not even closed: $\mathcal{L}_X \omega = -\gamma\omega$ and $(\Phi^t)^*\omega = e^{-\gamma t}\omega$, a direct violation of Thm. 3.6.

Proposition 6.4 (No invariant area form near the origin)**.** *For the field of Ctrex. 6.3 there is no smooth $\rho > 0$ on a neighbourhood $U \ni 0$ with $\mathcal{L}_X(\rho\,\omega) = 0$.*

Proof. Let $\mu = \rho\,\omega$ be invariant on U and $V = \frac{1}{2}(kq^2 + p^2/m)$, so $\dot{V} \leq 0$ and the sublevel sets $E_c = \{V \leq c\}$ are forward invariant and lie in U for small c, whence $\mu(\Phi^t(E_c)) = \mu(E_c)$ for $t \geq 0$. But $(\Phi^t(E_c))_{t \geq 0}$ is a decreasing family of compacta with intersection $\{0\}$, since by LaSalle the largest invariant subset of $\{p = 0\}$ is $\{0\}$; continuity from above gives $\mu(E_c) = 0$, contradicting $\rho > 0$. $\square$

Remark 6.5 (The honest caveat)**.** Proposition 6.4 must not be inflated into "the damped oscillator has no Lagrangian or Hamiltonian description", which is false: Caldirola–Kanai (at the price that H is not the energy, so ι silently changes), Bateman [3] (an unobserved degree of freedom) and conformally Hamiltonian (Liouville invariance) all work, so the claim is about the *typing* only. The inverse problem is solved completely only for two degrees of freedom [6], where systems with *no* variational multiplier exist; for $n \geq 3$ it is open [12].

Established physical result 6.6 (Nonholonomic constraints break Jacobi; cited)**.** For a constant-rank constraint distribution $\mathcal{D} \subset TQ$ the projected bracket is bilinear, antisymmetric and Leibniz—an *almost*-Poisson bracket—but its Jacobiator is tensorial in the curvature of $\mathcal{D}$ and vanishes *iff* $\mathcal{D}$ is integrable, i.e. iff the constraint is holonomic in disguise [4]. So Cor. 3.4 fails and the Liouville measure is generally not preserved.

7 Limits and degenerations

Proposition 7.1 (Explicit finite-time blow-up)**.** *On $T^*\mathbb{R}$ with $H = \frac{p^2}{2} - q^4$, the maximal integral curve through $(q_0, \sqrt{2}\, q_0^2)$, $q_0 > 0$, exists only on $(-\infty, t_*)$, $t_* = 1/(\sqrt{2}\, q_0) < \infty$.*

Proof. $\dot{q} = p$, $\dot{p} = 4q^3$; on $E = 0$ with $q > 0$, $p = \sqrt{2}q^2$, so $\dot{q} = \sqrt{2}q^2$ and $q(t) = (q_0^{-1} - \sqrt{2}t)^{-1} \to \infty$ as $t \uparrow t_*$. Compact level sets would suffice for completeness; here they are unbounded. $\square$

Established physical result 7.2 (Incompleteness is physical, not pathological)**.** The Newtonian n-body problem admits non-collision singularities; Xia proved existence for $n = 5$, settling Painlevé's problem [18], so bodies are ejected to infinity in finite time under the most canonical Hamiltonian in physics. Assumption 2.4 is therefore substantive: without it the processes of Def. 2.5 form a local groupoid. Time dependence costs as much, its repairs either leaving Def. 2.1 or adding an unobservable conjugate energy.

A *canonical relation* is a Lagrangian $L \subseteq \bar{M}_1 \times M_2$, $\bar{M} = (M, -\omega)$ [14], composed set-theoretically; call the result $\mathsf{SympRel}$. Weinstein's scare quotes are his own [15]: composition is defined only where the L_i meet *cleanly* and $F = L_1 \times_{M_2} L_2$ projects properly with connected fibres.

Proposition 7.3 (Four explicit failure modes)**.** *Take $M_i = \mathbb{R}^2$, where every curve is Lagrangian, and $L_1 = \Lambda_1 \times \Lambda_2$, $L_2 = \Lambda_2' \times \Lambda_3$ with all Λ curves, so the expected* $\dim F$ *is $4+6-8 = 2$. Then: (1)* excess—*$\Lambda_2' = \Lambda_2$ gives* $\dim F = 3$*, clean but not transverse, $e = 1$; (2)* non-injective projection—*$\Lambda_2 \pitchfork \Lambda_2'$ in k points gives* $\dim F = 2$ *but* $\mathrm{pr}\,|_F$ *is k-to-1, so composition forgets k; (3)* non-manifold fibre product—*$\Lambda_2 = \{p = 0\}$ and $\Lambda_2' = \{p = f(q)\}$ with $f(q) = e^{-1/q^2}$ for $q > 0$ and $f \equiv 0$ for $q \leq 0$ give $\Lambda_2 \cap \Lambda_2' = \{q \leq 0\}$, not a submanifold, so cleanness fails; (4)* emptiness—*for L_i graphs of symplectomorphisms on open sets, $L_2 \circ L_1$ can be empty.*

Proof. Direct verification of the dimension counts and intersections; (3) uses that f is C^∞ but not analytic. We do *not* exhibit a pair whose composite is nonempty yet fails to be an immersed Lagrangian: the literature asserts this can occur, we did not construct it, and nothing here relies on it. $\square$

New result claimed by this work 7.4 (The two-composition asymmetry)**.** $\boxtimes$ (Def. 2.5) is **total** and symmetric monoidal, but H is additive only absent interaction (4.1); relational interconnection in $\mathsf{SympRel}$ is **partial** (Prop. 7.3). This asymmetry—total kinematic composition, partial dynamical interconnection—is the sharpest constraint the finite-dimensional case places on the founding thesis.

8 Hidden assumptions and falsification

Hidden-assumption audit, each an Assumption, never a theorem, with what breaks if dropped. (H1) *Smoothness, finite even dimension*: impacts and dry friction leave the type. (H2) *Hausdorff, second countable*: no bump function in Prop. 3.1, yet reduction makes non-Hausdorff quotients. (H3) *Autonomous, closed, complete*: incompleteness makes "process" partial (Prop. 7.1) and every laboratory system is open. (H4) *Global ω, exactness*: "locally Hamiltonian" is strictly weaker (Rmk. 3.8), some manifolds are not symplectic (Prop. 6.2), and exactness supplies the polarisation (Lem. 5.3). (H5) $\mathcal{O} = C^\infty(M, \mathbb{R})$: algebraic closure, not empirical warrant—restricting to measurable quantities destroys the bracket (8.5). (H6) *Units live in* ι, u: $[\omega]$ is action, so a dimensionally inadmissible symplectomorphism is no physical equivalence. (H7) *Composition = Cartesian product*: this *is* the non-interacting case (4.1); real interconnection shares variables [16]. **The largest hidden assumption in the programme.** (H8) *Symplectomorphism = physical equivalence; a chart is a procedure*: the first is refuted here (5.1, 5.2); for the second, Darboux [9] makes infinitely many charts canonical, so the chart-to-procedure map is extra structure carrying the empirical content.

Empirical statement 8.1 (Dissipation)**.** Damped mechanical oscillators approach rest asymptotically: the measured phase portrait has an attracting fixed point, hence contracts area.

By Thm. 3.6 a Hamiltonian system has no attractors, so 8.1 falsifies the typing of Ctrex. 6.3, subject to Rmk. 6.5.

Empirical statement 8.2 (Nonholonomic systems)**.** A rolling disc and a Chaplygin sleigh conserve energy yet are observed to have asymptotically stable motions and not to preserve phase-space volume, so by 6.6 finite-dimensional classical mechanics is *not* coextensive with symplectic-or-Poisson geometry. In the thesis's favour, Lagrange–d'Alembert and vakonomic mechanics give *different* equations for one constraint set and experiment selects d'Alembert.

Empirical statement 8.3 (Finite data)**.** Measured trajectories are finite-precision samples: observables in $\mathcal{O}$ are read to precision $\varepsilon > 0$ over a finite horizon T.

New result claimed by this work 8.4 (Observational underdetermination of the bare object)**.** The map from interpreted objects to finite-precision data is many-to-one, and five mechanisms are exact, not approximate: (i) (M, ω, H) and $(M, c\omega, cH)$ have the *identical* vector field; (ii) H and $H + \text{const}$ are indistinguishable; (iii) on a level set H and $f(H)$ give the same orbits up to reparametrisation; (iv) symplectomorphisms mix q and p, so "position" is fixed by exactness and ι, not by (M, ω) (Lem. 5.3); (v) one second-order system can carry inequivalent Hamiltonian structures [6, 12]. With 8.3 a sixth, approximate mechanism appears: data of precision ε over horizon T cannot separate an integrable H from a KAM-perturbed neighbour.

Established physical result 8.5 (Groenewold–van Hove, in correct scope)**.** No map from the Poisson algebra of *all* polynomials on $\mathbb{R}^{2n}$ to self-adjoint operators is simultaneously linear, sends $\{f, g\}$ to $\frac{i}{\hbar}[Q(f), Q(g)]$ and 1 to I, and is irreducible on $Q(q^i), Q(p_j)$; the obstruction appears at degree four [7, 13]. It does *not* make quantisation impossible: it kills full-algebra coverage *together with* irreducibility—a constraint on the observable slot, not on mechanics.

Established physical result 8.6 (Ostrogradsky; nondegeneracy is load-bearing)**.** For a *nondegenerate* Lagrangian with derivatives to order $N \geq 2$ the canonical Hamiltonian is linear in at least one momentum, hence unbounded below [17]. It does *not* say higher-derivative theories are inconsistent: degenerate ones evade it legitimately.

Established physical result 8.7 (Gauge systems: the symplectic type is simply wrong)**.** Degenerate Lagrangians give a non-invertible Legendre map onto a constraint surface carrying a presymplectic form, on which $\iota_X \omega = dH$ is neither always solvable nor single-valued; dynamics is defined only after the Dirac–Bergmann algorithm terminates, the physical object being a quotient by first-class gauge orbits [5]. For a reparametrisation-invariant system $H \approx 0$ there, so "evolution is the flow of H" becomes pure gauge—the finite-dimensional germ of the problem of time.

Established physical result 8.8 (Boundary test: quantum mechanics *satisfies* the symplectic type)**.** For finite-dimensional quantum systems the pure-state space $\mathbb{CP}^{n-1}$ is symplectic (indeed Kähler) and Schrödinger evolution *is* the Hamiltonian flow of $H(\psi) = \langle\psi|\widehat{H}|\psi\rangle$ [8].

Interpretation 8.9 (Why 8.8 damages rather than rescues the thesis)**.** The claim that quantum mechanics refutes the symplectic type is false, and the truth is worse: satisfying the type does not pin down the physics. What distinguishes quantum mechanics is structure the type lacks—the Fubini–Study metric carrying the probabilistic content, the restriction to unitary flows (generic Hamiltonian flows on $\mathbb{CP}^{n-1}$ are not quantum), mixed states, and measurement maps, which are not symplectomorphisms. General relativity and quantum field theory sit outside the type for infinite dimension, GR also for $H \approx 0$. Hamiltonian flow is also time-reversal symmetric and measure preserving, so macroscopic irreversibility cannot be derived from the object alone.

New result claimed by this work 8.10 (Falsification bookkeeping)**.** The object is (i) *not universal*—it fails for 8.1, 8.2, irreversible systems, gauge systems (8.7) and all field theory; (ii) *not sufficient*—8.8 with Interp. 8.9; (iii) *not determined by data*—8.4; (iv) *not total*—Prop. 7.1, 7.2; (v) *not compositional in the assumed sense*—4.1, 7.4. What it *is*: an accurate description of closed, smooth, holonomically constrained, finite-dimensional conservative systems, with an exact account of its own equivalence classes; we claim that and nothing broader. Repairing the composition defect means importing behavioural interconnection [16], yielding a *different* theory object.

9 Source status

Two research lines ran independently and diverged on five points, each resolved by weakening a claim; their agreement elsewhere is convergent evidence, never proof. *Cross-checked by exact rational polynomial arithmetic during drafting, with a non-closed control form*: Thm. 3.3 *including its sign*, and the Kepler brackets with signs and both syzygies. Those check scripts are not shipped in this bundle and carry no receipt, so they are corroboration only; the load-bearing support for each is the derivation given in the text. *Not verified, therefore not asserted*: the Kirillov–Kostant–Souriau normalisation; any record for the algebra-reconstruction theorem, so no converse to the isomorphism criterion is claimed; Weinstein's 2010 *Symplectic categories* and Wehrheim–Woodward; Ostrogradsky's 1850 memoir, for which [17] stands in; the primary Kirillov and Kostant papers; the contact-thermodynamics and metriplectic literatures, so Interp. 8.9's irreversibility remark rests on Thm. 3.6 alone; and [13], two 1951 items being routinely conflated, with [12], whose series designation could not be pinned down.

10 Conclusion

The typing is most informative where it fails: interconnection is not composition; the Cartesian product composes kinematics totally but dynamics only absent interaction (4.1, 7.4); isomorphism underdetermines physical identity (5.1, 5.2); and the process slot is not total (7.2). The thesis survives as a research programme and fails as stated (Conjecture 4.2).

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